THE MIXING TANK PROBLEM

Mixing tank problems are a subset of a generalized balance equation seen here.

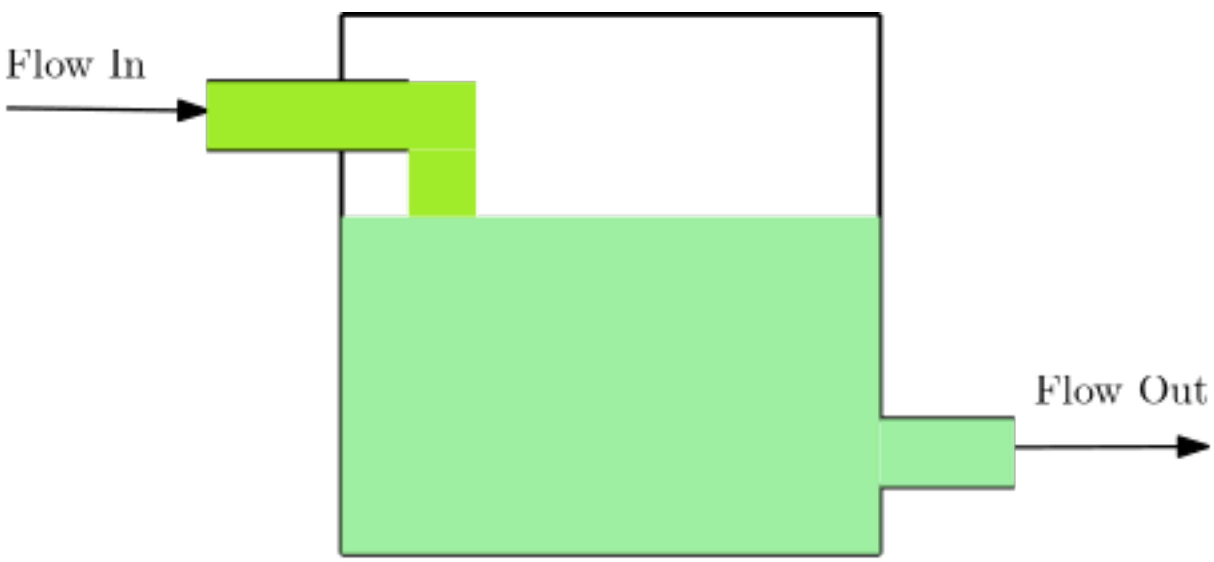
$$Accumulation = Input - Output + Generation - Consumption.$$

We won't be solving any chemical engineering problems, so we will let *Generation* and *Consumption* equal zero.

$$Accumulation = Input - Output$$

We will also literally put a box around the system of interest so we can define the ins and outs. Here, the system will consist of one or more tanks of some specified volume. The tank will have a flow in and a flow out. What specifically are we interested in?

We are interested in the change in the amount of something dissolved in the fluid. It is conceptually easy to think of the amount of salt dissolved in the fluid.



Now, it is the rate of change of salt present in the tank that we want to understand. Let the amount of salt dissolved in the tank at any point in time be $A(t)$. We are then interested in the rate of change of A or

$$\frac{d}{dt}A(t)$$

The let the *Input* be the rate at which salt enters the tank and the *Output* be the rate at which salt leaves the tank.

How should "Rate at which $A(t)$ enters the tank" be defined? When salt is dissolved in water its concentration is defined as

$$\frac{\textit{Amount of salt present at any time t}}{\textit{Volume of water any time t}}$$

So if we multiply concentration by flow or $\frac{\textit{Volume}}{\textit{time t}}$ we get

$$\frac{\textit{Amount of salt present at any time t}\cdot 1}{\textit{Volume of water any time t}} X \frac{\textit{Volume}}{\textit{time t}} = \frac{\textit{Amount of salt}}{\textit{time}}$$

Now we can write the final equation as

$$\textit{Rate of change of } A(t) = \textit{Rate } A(t) \textit{ enters tank} - \textit{Rate } A(t) \textit{ exits tank}$$

$$\textit{Rate of change of } A(t) = \frac{\mathrm{d}}{\mathrm{d}t}A(t)$$

$$\textit{Rate } A(t) \textit{ enters tank } = (\textit{flow rate of liquid entering tank}) \cdot (\textit{concentration of substance entering})$$

$$\textit{Rate } A(t) \textit{ exits tank } = (\textit{flow rate of liquid exiting tank}) \cdot (\textit{concentration of substance exiting})$$

Initial conditions will be specified in the problem statement. There is usually a tank of a certain volume with some amount of substance dissolved in that volume, e.g 10 kg of salt in 1000 liters. The must be an input flow with known concentration of substance. This input concentration is known and can be zero or can be constant or even time-varying. There is usually an output flow which can be different than the input flow. If the input and output flows are not equal, then the volume of fluid in the tank will change as a function of time. Finally, we are solving for the amount of substance dissolved in the tank at any time.

Example 1 Parking Lot Runoff

A 150 space parking lot has dimensions 242' x 180'. In the winter when freezing rain is forecast, dry salt is applied at a rate of 3 lbs per 1,000 square foot area. Assume that all of this salt ends up in a nearby, half acre spring-fed pond with a flow rate of 10 gal/min. Assume the pond depth is 2 feet with the parking lot runoff. How long will it take to clear the salt from the pond assuming pond outlow is the same as the inflow from the spring?

To start, let's find the amount of salt that will end up in the pond.

restart

$lot := 242 \cdot 180;$

$$lot := 43560 \qquad (1)$$

So, the lot is exactly 1 acre in size. The amount of salt in lbs is then

$salt := evalf\left(\frac{(lot \cdot 3)}{1000}\right);$

$$salt := 130.6800000 \qquad (2)$$

Next we need the volume of the pond in cubic feet with the runoff and salt.

$V\_pond := \frac{lot}{2} \cdot 2;$

$$V\_pond := 43560 \qquad (3)$$

We have the volume in cubic feet. How many gallons is that since our flow rate is in gallons per minute.

1 ft3 = 7.48052 gallons

$V\_gallons := V\_pond \cdot 7.48052;$

$$V\_gallons := 325851.4512 \qquad (4)$$

Let's also convert gallons/min to gallons/day.

$flow := 10 \cdot 1440;$

$$flow := 14400 \qquad (5)$$

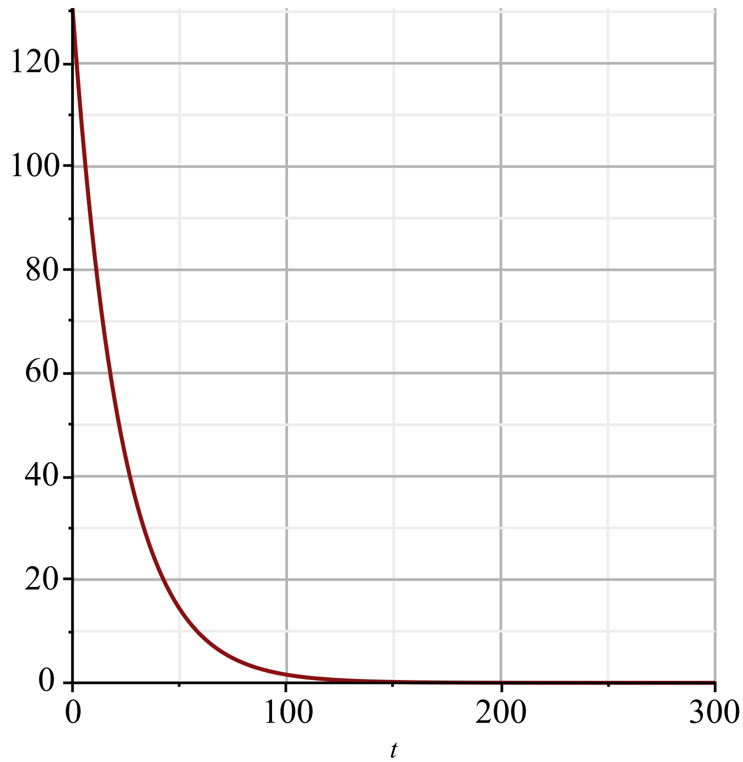
$clear := \frac{d}{dt} S(t) = 10 \cdot 0 - \frac{flow \cdot S(t)}{V\_gallons};$

$$clear := \frac{d}{dt} S(t) = -0.04419191612\, S(t) \qquad (6)$$

$Ans1 := evalf(rhs(dsolve(\{clear, S(0) = salt\})));$

$$Ans1 := 130.6800000\, e^{-0.04419191612\, t} \qquad (7)$$

$plot(Ans1(t), t=0..300, size=[300, 300], gridlines)$



Let's see how much salt is left at day 200.

$$\left. Ans1 \right|_{t=200}$$

$$0.01895605948 \qquad (8)$$

Note that the level isn't even 1% of the starting salt. A deeper pond with this modest flow from the spring would take even longer. It would seem like a good idea to divert the parking lot runoff to a catch basin since the salt lingers so long in the pond.